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In this paper we examine gas flow in a porous medium with porosity discontinuity surfaces. The conditions on such surfaces are obtained by invoking additional assumptions within the scope of the models described in [1]. As an illustration, we present the solution of a problem of nonstationary flow in a pipe with a porous insert with finite width. We note that in works on gas flows in porous media, porosity discontinuities are not usually adequately studied. For example, continuity conditions, which can be reasonably justified only for low subsonic velocities, are imposed on them. In this sense, [2] is an exception. In [2], the assumption of conservation of entropy during gas inflow into a porous material and the "Borda impact" scheme when the gas flows out of it was used for problems involving the interaction of a shock wave with a porous half-space and with a porous coating. The system of assumptions in [1], adopted for the same purposes in what follows, including the schemes indicated, permits analyzing a wider class of regimes (for example, sonic or supersonic gas flow out of a porous material into a region with sufficiently low pressure).

1. Since in what follows the main attention is devoted to porosity discontinuities, we shall restrict ourselves to a quite simple model of a porous medium, referred to in what follows as the material. We shall assume that the material is absolutely rigid, that its properties at each point do not depend on orientation (isotropy), and that there are no so-called blind pores. In addition, we shall assume that viscosity and thermal conductivity are important only in processes of force and thermal interaction between the gas and the material. Without specifying other assumptions, widely used in the theory of percolation of gases and liquids (see [3, 4]), we shall write out the integral conservation laws for mass, momentum, and energy of the gas and the energy of the material. If  $t$  is the time and  $V$  is an arbitrary volume, occupied by the gas and material (the latter can also be absent), fixed in space, and bounded by the surface  $\partial V$ , then the laws listed above have the form

$$\begin{aligned} \int_V \int m \rho dV \Big|_{t_0}^t &= - \int_{t_0}^t dt \int_{\partial V} m \rho v_n d\sigma, \\ \int_V \int m \rho v dV \Big|_{t_0}^t &= - \int_{t_0}^t dt \int_{\partial V} m (p \mathbf{n} + \rho v_n \mathbf{v}) d\sigma + \int_{t_0}^t dt \int_V \int \mathbf{F} dV, \\ \int_V \int m \rho \left( e + \frac{v^2}{2} \right) dV \Big|_{t_0}^t &= - \int_{t_0}^t dt \int_{\partial V} m \rho v_n \left( i + \frac{v^2}{2} \right) d\sigma + \int_{t_0}^t dt \int_V \int m (1 - m) \rho q dV; \\ \int_V \int (1 - m) E^p dV \Big|_{t_0}^t &= - \int_{t_0}^t dt \int_V \int m (1 - m) \rho q dV, \end{aligned} \tag{1.1}$$

where  $d\sigma$  is an element of  $\partial V$  with the unit external normal vector  $\mathbf{n}$ ;  $m$  is the porosity (relative pore volume);  $\rho$  is the density;  $p$  is the pressure;  $e$  is the specific (per unit mass) internal energy, which is a known function of  $p$  and  $\rho$ ;  $i = e + p/\rho$  is the specific enthalpy;  $\mathbf{v}$  is the velocity of the gas;  $v = |\mathbf{v}|$ ;  $v_n = \mathbf{v} \cdot \mathbf{n}$ ;  $E^p$  is the energy per unit volume of material;  $\mathbf{F}$  is the force with which the material, located in a unit volume of the medium, acts on the gas;  $m(1 - m)\rho q$  is the heat flux out of this material into the gas (we have in mind the gas

located in the same volume). The factor  $m(1 - m)\rho$  in front of  $q$  is introduced for convenience and  $t$  and  $t_0$  are arbitrary times ( $t > t_0$ ).

In order to close system (1.1), together with an equation of state of the gas  $e = e(p, \rho)$  or the equations  $e = e(p, T)$  and  $\rho = \rho(p, T)$  and the corresponding equation  $E^P = E^P(T^P)$  for the material ( $T$  and  $T^P$  are the temperature of the gas and of the material), expressions are required for  $F$  and  $q$ . The expressions indicated are usually taken in the form

$$F = p\nabla m - m(1 - m)\rho f, \quad f = \varphi_f v, \quad q = \varphi_q(T^P - T), \quad (1.2)$$

and in addition  $\varphi_f$  and  $\varphi_q$  are known positive-definite functions of scalar parameters of the gas and of the material (including  $v$ ), but not their derivatives. The factor in front of  $f$  is introduced in (1.2) from the same considerations as in front of  $q$  in (1.1). In the equation for  $F$ , the first term gives the force stemming from the change in the "throughput" section of pores. Its meaning is easy to understand by comparing (1.1) and (1.2) with the equations for quasi-one-dimensional flow in a channel with a variable area  $S$ , in which the force acting on the gas from the side of the walls in the segment of the channel where  $S$  changes by  $\Delta S$  equals  $p\Delta S$ . The second term in the same equation is related to the friction between the gas and the material. For low velocities  $\varphi_f$  does not depend on  $v$ , and for a uniform material ( $\nabla m \equiv 0$ ) the first equation in (1.2) reduces to Darcy's law:  $F = -kv$  with a positive constant  $k$ . According to (1.1) and (1.2),  $(1-m)f$  and  $(1-m)q$  are quantities referred to unit mass of gas.

Before going on to discontinuity surfaces for  $m$ , on which  $\nabla m$  becomes infinite and, for this reason, the equation for  $F$  becomes meaningless, we shall present for completeness the equations for the flow following from (1.1) and (1.2), valid in subregions of continuity of all parameters, and relations on strong discontinuities, different from the discontinuity surfaces of the properties of the material. These equations and relations are obtained from (1.1) and (1.2) by a standard method and reduce to well-known differential and finite equalities:

$$\begin{aligned} m \frac{\partial \rho}{\partial t} + \nabla(m\rho v) &= 0, \quad \frac{dv}{dt} + \frac{1}{\rho} \nabla p + (1 - m) f = 0, \\ T \frac{ds}{dt} - \frac{di}{dt} - \frac{1}{\rho} \frac{dp}{dt} &= (1 - m)(q - vf) \left( \frac{d}{dt} = \frac{\partial}{\partial t} + v\nabla \right), \\ \frac{\partial E^P}{\partial t} = -m\rho q, \quad [\rho(v_n - D)] &= 0, \quad [p + \rho(v_n - D)^2] = 0, \\ \rho(v_n - D) v_\tau &= 0, \quad [\rho(v_n - D) \{2i + (v_n - D)^2\}] = 0, \quad D[E^P] = 0 \end{aligned} \quad (1.3)$$

In obtaining these relations, it was assumed that  $m$  does not depend on  $t$ . In (1.3),  $d/dt$  is the total differential operator with respect to  $t$  along the trajectory of a gas particle;  $s$  is the specific entropy of the gas, and, in addition, it was assumed that  $s = s(i, p)$  and  $Tds = di - (1/\rho)dp$ ; the square brackets indicate the difference of the quantities enclosed on the discontinuity;  $v_n$  is the projection of  $v$  along the vector normal  $n$  to the discontinuity;  $D$  is the velocity discontinuity along the normal to itself and  $D > 0$ , if the discontinuity moves in the direction  $n$ ;  $v_\tau$  is the projection of  $v$  tangent to the discontinuity. On the strength of the last condition in (1.3),  $E^P$  or  $T^P$  can be discontinuous only on discontinuities that do not move along the material. We also note that according to (1.2) and the fourth equation in (1.3), stationary flows are possible, strictly speaking, within the scope of the model described only when the temperatures of the gas and material are equal ( $T^P = T$ ). If, as often happens, the heat capacity of the material is much greater than the heat capacity of the gas, the rate of change of  $T^P$  turns out to be so small that the flow is practically stationary for  $T^P \neq T$  as well. The thermal conductivity of the material, which often exceeds by orders of magnitude the thermal conductivity of the gas and in many cases must be taken into account in constructing a model, acts in the same direction. Taking into account the thermal conductivity of the material leads to a change in the fourth and eliminates the last equation in system (1.3). Naturally, in this case, discontinuities in  $E^P$  become impossible. In what follows, in the problem of gas flow through a porous insert (Sec. 3), instead of the equation for  $E^P$  from (1.1) or its result from (1.3), the material is assumed to be isothermal ( $T^P \equiv \text{const}$ ).

2. On the porosity discontinuity surface  $\partial V^m$ , the component of the gradient  $\partial V^m$  normal to  $\nabla^m$  becomes infinite, while on the strength of (1.2), the corresponding component of the

force  $F$  also becomes infinite. The divergence of some volume quantity (in this case the force components), as usual, indicates the necessity for introducing an analogous surface characteristic, which we shall denote by  $F^\sigma$ . It is natural to assume that

$$F^\sigma = p^\sigma [m]. \quad (2.1)$$

Here, as previously,  $[m] = m_+ - m_-$ ; the plus (minus) index is assigned to parameters before (after) the discontinuity in the direction of flow; the normal  $n$  to  $\partial V^m$  is chosen so that  $v_n = v \cdot n \geq 0$ ;  $F^\sigma$  is the projection of the surface force on  $n$ ;  $p^\sigma$  is the average pressure, acting on the end-face segments of the porous material. Since in order to determine  $p^\sigma$  it is necessary to make additional assumptions, (2.1) should be viewed, for now, as a relation between two as yet unknown quantities;  $F^\sigma$  and  $p^\sigma$ .

Taking into account in (1.1) the presence of a surface force on  $\partial V^m$  and following the usual procedure in other respects, we find that within the scope of the model of an isotropic material adopted, the following equalities must be satisfied on the porosity discontinuities:

$$\begin{aligned} [m\rho v_n] &= 0, [m(p + \rho v_n^2)] = F^\sigma, \\ [m\rho v_n v_\tau] &= 0, [m\rho v_n(2i + v^2)] = 0. \end{aligned} \quad (2.2)$$

The basic difference between these equalities and similar relations in (1.3), written for stationary discontinuities ( $D = 0$ ), lies in the presence of the factor  $m$  and the surface force  $F^\sigma$  in (2.2). In addition, in order that the last condition in (2.2) be applicable for models that are different from the one being examined here, the term  $v^2$  in it is retained, although on the strength of the first and third conditions,  $v^2$  can be replaced by  $v_n^2$ , as is done in (1.3).

Since  $F^\sigma$  in (2.2) or  $p^\sigma$  in (2.1) are as yet unknown, in order to close the system of conditions on the discontinuity  $m$ , additional assumptions are necessary. In addition, it is necessary to distinguish three possibilities: 1)  $j \equiv (\rho v_n)_+ = (\rho v_n)_- = 0$ , i.e., the gas does not flow through the discontinuity at the point being examined; 2) the gas flows into a region with lower porosity; 3) the gas flows out of this region. If  $j = 0$ , then, as for the usual tangential discontinuity in the gas, all conditions (2.2), except the second condition, are satisfied for arbitrary jumps in  $v_\tau$  and total entropy  $I = i + v^2/2 = i + v_\tau^2/2$ . The remaining condition in (2.2), which for  $j = 0$  reduces to

$$[mp] = p^\sigma [m], \quad (2.3)$$

is satisfied if  $p^\sigma = p_+ = p_-$ . Therefore, it may be assumed that those regions of the porosity discontinuity surfaces through which gas does not flow do not differ in the model being examined from the usual tangential discontinuities. The same result is obtained from (2.3), if instead of assuming that the three pressures ( $p^\sigma$ ,  $p_+$ , and  $p_-$ ) coincide, a weaker assumption, that  $p^\sigma = \alpha p_+ + \beta p_-$  with  $\alpha + \beta = 1$  and  $0 \leq \alpha \leq 1$ , is used.

If the gas flows into the region with lower porosity, then we shall limit ourselves to the case of subsonic normal inflow velocity components ( $M_{n-} < 1$ , where  $M_{n-} = v_n/\alpha$ , while  $\alpha$  is the velocity of sound in the gas), which is completely sufficient for the assumptions encountered most often. Since here the inflow occurs with an increase in velocity and with a drop in pressure, as in [1, 2], it is natural to assume that in this case there are no significant separations, accompanied by an increase in the gas entropy and therefore

$$[s] = 0. \quad (2.4)$$

Here  $s$  can be taken as any function of the gas entropy, for example,  $pp^{-\kappa}$  for a perfect gas with an adiabatic index  $\kappa$ . Condition (2.4) together with the first, third, and fourth equalities in (2.2) forms a closed system. Solving it, it is then possible to determine in each specific problem, if necessary,  $F^\sigma$  from the second equation in (2.2), and  $p^\sigma$  from  $F^\sigma$  using (2.1).

In the case in which the gas flows out of the region with lower porosity, according to [1], we shall examine three different regimes. If  $M_{n\pm} < 1$ , i.e., for a completely subsonic outflow, the expansion of the flow is accompanied by an increase in pressure, which does not permit using the assumption of an absence of separations and conservation of entropy. Turning in this case to the "Borda impact" discontinuity scheme we set

$$p^\sigma = p_-, \quad (2.5)$$

which together with (2.1) closes the system (2.2).

For sonic and supersonic outflow ( $M_{n-} \geq 1$ ), condition (2.5) is not satisfied, while the flow, passing through the porosity discontinuity, can become both sub- and supersonic. Since all quantities in (2.2) with the minus index are known for  $M_{n-} \geq 1$ , for  $M_{n+} < 1$ , the parameters with the plus index, just as in [1], are determined from a solution of the problem of flow in the region with high porosity using only three (the first, third, and fourth) conditions from (2.2). The second condition in (2.2), together with (2.1), can, as in the case of gas inflow into a region with low porosity, then be used for calculating  $F^\sigma$  and  $p^\sigma$ . Retardation from  $M_{n-} \nearrow 1$  to  $M_{n+} < 1$  occurs under the action of jolts (oblique and almost straight, "closing") and mixing on boundaries of jets and separation zones. In addition, as in other cases, the microstructure of the flow is important only in a layer, whose thickness along  $n$  is of the order of the characteristic size of pores (or the distance between them) and for this reason comprises the internal structure of the discontinuity.

In contrast to the preceding case, for supersonic flow from both sides of the discontinuity ( $M_{n-} \geq 1$ ,  $M_{n+} > 1$ ), all parameters with the plus index must be determined uniquely by parameters with the minus index. The latter, according to [1], is achieved by adding to (2.1) and (2.2) either condition (2.4) or the equality

$$p^\sigma = p_+, \quad (2.6)$$

and, in addition, in light of the comparison carried out in [1] with results of experiments on the interaction of shock waves with perforated barriers, the use of (2.6) is preferable.

We emphasize that relations (2.1)-(2.6), like the equations of Sec. 1, describe gas flow in an isotropic material, which also has a series of other properties specified previously. The difference between the real properties of the material and the assumed properties makes it necessary to make corresponding corrections. We shall illustrate this for a material with "limiting anisotropy," which we shall take to mean the following. Let the velocity vector be written in the form  $v = v_l$ , where  $l$  is a unit vector, defining the orientation of  $v$ . For a limiting anisotropic material, we will assume that  $l$  is a known characteristic, which can be discontinuous. Without considering all the changes necessary in such a case, we shall consider two points. First of all, since now  $l$  is a known function of the coordinates, instead of three vector equations of motion only a single equation from (1.3) remains:

$$\frac{dv}{dt} + \frac{1}{\rho} \frac{\partial p}{\partial l} + (1-m) f^l = 0 \quad \left( \frac{d}{dt} = \frac{\partial}{\partial t} + v \frac{\partial}{\partial l}, f^l = f \cdot l = \varphi_j v^j \right),$$

where  $\partial/\partial l$  is the operator of differentiation along the coordinate, measured along the streamline. Second, on the discontinuity surfaces for  $m$  and  $l$ , the condition for the component of the velocity  $v_\tau$  tangent to the discontinuity from (2.2) is replaced by

$$[m \rho v_n v_\tau] = F_\tau^\sigma, \quad (2.7)$$

where  $F_\tau^\sigma$  is a component of the surface force, introduced in an appropriate manner. The remaining conditions from (2.2), in this case, remain as before (the fourth, due to the fact that in it  $v^2$  is not replaced by  $v_n^2$ ).

For a limiting anisotropic material,  $l_\pm$  are known on both sides of the discontinuity and (2.7) is used only for calculating  $F_\tau^\sigma$ . A similar situation occurs also for an inflow of gas into an anisotropic material from an isotropic material. However, the situation changes if the gas flows out of the anisotropic material into the isotropic material (or in a space free of the material). Here  $F_\tau^\sigma$  must be given. In particular, we can set  $F_\tau^\sigma = 0$ , which for  $j \neq 0$  leads to conservation of  $v_\tau$ .

Without analyzing other properties of the flow in different anisotropic media, we note that, as evident from the preceding discussion, obtaining conditions on a discontinuity of the type studied requires an examination of the microstructure of the flow in some neighborhood of the discontinuity. Although such an analysis is possible at different levels (using various assumptions, as done above, or in the approximation of a more complete model with a microscopic analysis of the structure of the material, taking into account the gas viscosity, etc.), closure of the corresponding system of relations on the discontinuity without this, as a rule, is impossible. A similar situation occurs not only in percolation problems, but also in studying the flow of gas through perforated barriers, boundaries of a free space, an infinitely dense lattice of profiles, etc., always when the discontinuity replaces



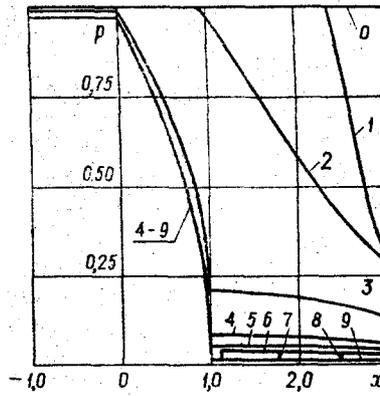


Fig. 2

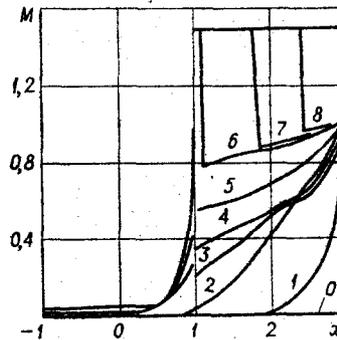


Fig. 3

0.02), 2 ( $t = 0.08$ ), 3 ( $t = 0.22$ ), 4 ( $t = 0.45$ ), 5 ( $t = 0.66$ ), 6 ( $t = 1.00$ ), 7 ( $t = 1.22$ ), 8 ( $t = 1.55$ ), 9 ( $t = 2.00$ ). The time is referred to  $l^0/a_0^0$  and the pressure to  $\rho_0^0 a_0^0{}^2$ ; a 0 upper index indicates a dimensional quantity;  $a_0^0 = \sqrt{\kappa p_0^0/\rho_0^0}$  is the initial velocity of sound in the gas. Curves 1-4 correspond to subcritical flow out of the material ( $M_- < 1$ ), which occurs in the Borda impact regime. Curve 5 corresponds to critical outflow ( $M_- = 1$ ) with  $M_+ < 1$ . By this time, the flow in the insert is practically established. Within the scope of the model adopted, in the regime indicated, the microstructure of the efflux zone of the jets is terminated by a stationary closing jump. After the pressure to the right of the closing jump, as a result of evacuation of the end segment of the pipe ( $1 < x < X$ ), becomes sufficiently low, while the intensity of the jump decreases (the pressure to the left of the jump remains practically the same), a regime with  $M_+ > 1$  is realized. At the same time, the closing jump is displaced to the right (curves 6-9) and at some time is carried out of the pipe.

The results, presented in Figs. 2 and 3, were obtained assuming that the material is isothermal ( $TP \equiv T_0$ ) for a perfect gas with a gas constant  $R^0$ . The functions  $\varphi_f^0$  and  $\varphi_q^0$ , as in the case of Darcy's law, were assumed to be constant. It can be shown that in this case the solution in dimensionless variables (the density is scaled to  $\rho_0^0$ , the velocity to  $a_0^0$ , the enthalpy to  $a_0^0{}^2$ , and the temperature to  $a_0^0{}^2/R^0$ ) depends on five dimensionless constants:  $\kappa$ ,  $X$ ,  $m$ ,  $k_f = (1-m)\varphi_f^0 l^0/a_0^0$ , and  $k_q = (1-m)(\kappa-1)\varphi_q^0 l^0/a_0^0$ . Figures 2 and 3 correspond to  $\kappa = 1$ , 4,  $X = 3$ ,  $m = 0.9$ ,  $k_f = 5$ , and  $k_q = 0.023$ .

The calculation was carried out using Godunov's difference scheme [6], modified for the problem being examined. Without considering all the details, we shall only indicate two points. First, a considerably nonuniform grid, whose cells decreased smoothly when the section  $x = 1$  was approached from both sides, was used. Second, an analogous problem concerning disintegration of a discontinuity upon the surface of the jump  $m$  was added to the problem of the disintegration of an arbitrary discontinuity in the gas, which, as is well known, constitutes the basic scheme proposed by Godunov. It was solved within the scope of the flow schemes described in Sec. 2. Naturally, the same schemes at each time relate the discontinuities in parameters at the boundaries of the insert (for  $x = 0$  and  $x = 1$ ).

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#### INTERACTION OF SHOCK WAVES AND PROTECTIVE SCREENS IN A LIQUID AND IN A TWO-PHASE MEDIUM

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The significant, for practical applications, characteristics of changes in shock-wave parameters at boundaries separating a two-phase mixture and a continuous liquid are clarified by investigations of the propagation of pressure waves in two-phase gas-liquid media.

One of the often-discussed practical applications of the study of the dynamics of wave processes in a two-phase medium is related to the damping of pressure waves by bubble screens. However, almost any problem with damping of pressure waves in a two-phase medium separates into two independent, but closely related, problems of their attenuation and amplification on boundaries separating media with different acoustical impedance. Thus, the problem of amplification of pressure waves is encountered in analyzing their transition into a medium with a high acoustical impedance. It is well known that when shock waves are incident on a separation boundary in a two-phase medium, as the acoustical impedance increases, the pressure differential on the shock front increases by a factor of up to 5-7 [1-5]. When shock waves pass into a medium with a lower acoustical impedance, the pressure waves are observed to attenuate by a factor of 3-5 [1-5] or damping of short wavelength excitations in the gas-fluid medium becomes possible.

In this connection, depending on the specific conditions, it turns out that protective properties of water bubble screens in liquids begin to be determined by the ratios of the acoustical impedances of the liquid and the two-phase medium on both boundaries of the screen; on one of them, the pressure differential increases, while on the other it decreases. Therefore, the effectiveness of screens will depend on the pressure in the medium, the volume concentration of gas in the liquid, and the intensity of the wave. When the acoustical impedances of the continuous liquid and the two-phase medium approach one another, as noted, for example, with an increase in pressure or decrease in the volume concentration of gas in the liquid, the protective screens become transparent to shock waves and, therefore, become ineffective.

Computational data, indicating the low effectiveness of bubble screens for damping shock waves with pressure differential on the front exceeding 5 mPa with the volume concentration of gas in the liquid up to 10%, were already obtained in [6]. However, the computed

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